

Online Appendix

Untangling the Role of Assortative Mating in Educational Reproduction in Twelve European Countries

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* This Online Appendix contains additional information regarding the article:
<https://www.comparativepopulationstudies.de/index.php/CPoS/article/view/604/423>.



Tab. A1: Descriptive statistics for the retrospective anchor sample (GGS)

Country samples		Respondent			Retrospective information on Mothers	
		Birth Year	Number of Siblings	High-Educated	Birth Year	High-Educated
Austria N=1,849	Mean	1970	2.4	.23	1942	.046
	SD	5.2	1.8	.42	5.3	.21
Belgium N=1,636	Mean	1966	2.1	.5	1939	.18
	SD	7.2	1.6	.5	6.2	.39
Bulgaria N=4,436	Mean	1966	1.3	.26	1941	.11
	SD	7.1	1.2	.44	6	.31
Czech Republic N=2,615	Mean	1965	1.4	.2	1941	.054
	SD	7.9	1	.4	6.2	.23
Georgia N= 3,414	Mean	1966	2.2	.33	1939	.15
	SD	7.5	1.5	.47	6	.36
Germany N=2,534	Mean	1965	1.8	.31	1939	.073
	SD	6.8	1.5	.46	5.7	.26
Lithuania N=2,429	Mean	1966	1.7	.28	1939	.1
	SD	7.8	1.4	.45	6.1	.3
The Netherlands N= 2,257	Mean	1967	2.1	.41	1940	.099
	SD	6.3	1.5	.49	6	.3
Poland N=4,858	Mean	1966	2.4	.23	1940	.062
	SD	8.4	1.8	.42	6.5	.24
Romania N=4,065	Mean	1964	2.1	.12	1939	.022
	SD	7.7	1.8	.33	6.2	.15
Russia N=1,962	Mean	1965	1.6	.48	1939	.25
	SD	7.3	1.4	.5	6	.43
Sweden N=2,126	Mean	1967	1.8	.43	1941	.35
	SD	7.8	1.1	.5	6.1	.48
Total N=34,181	Mean	1966	1.9	.29	1940	.11
	SD	7.5	1.6	.45	6.2	.31

Source: own calculation based on retrospective sample based on GGS data.

Tab. A2: Descriptive statistics for the pooled fertility dataset

Country samples		Birth Year	High-educated	childless	Number of children
Austria	Mean	1942	.16	.15	2.2
N= 1,832	SD	5.7	.37	.36	1.5
Belgium	Mean	1941	.24	.2	2
N= 2,607	SD	6	.42	.4	1.5
Bulgaria	Mean	1942	.17	.059	1.8
N=2,475	SD	5.9	.38	.24	.89
Czech Republic	Mean	1942	.092	.096	1.9
N=4,008	SD	5.8	.29	.29	1.1
Georgia	Mean	1942	.26	.23	1.8
N= 1,132	SD	5.8	.44	.42	1.4
Germany	Mean	1941	.17	.38	1.3
N=3,438	SD	5.8	.38	.49	1.4
Lithuania	Mean	1942	.17	.21	1.6
N=1,167	SD	5.5	.38	.41	1.3
The Netherlands	Mean	1942	.22	.17	2.2
N= 2,623	SD	5.7	.42	.37	1.4
Poland	Mean	1942	.12	.12	2.2
N=3,463	SD	6	.33	.32	1.5
Romania	Mean	1941	.062	.14	1.9
N=1,605	SD	6	.24	.35	1.4
Russia	Mean	1941	.38	.13	1.8
N=2,030	SD	6.1	.49	.33	1.1
Sweden	Mean	1942	.28	.096	2.3
N=2,617	SD	5.5	.45	.29	1.3
Total	Mean	1942	.19	.16	1.9
N=28,997	SD	5.9	.39	.37	1.3

Source: own calculation based on the pooled prospective dataset.

Tab. A3: Proportions of childless women and completed cohort fertility rate (CFR) of high- and low-educated women

Country	Cohort	Proportion of childless		Cohort fertility rate	
		High-educated	Low-educated	High-educated	Low-educated
Austria	1930-1935	.16	.11	1.4	2.2
	1936-1940	.21	.13	1.2	2.1
	1941-1945	.24	.14	1.3	2
	1946-1950	.31	.11	.98	1.8
	Total	.23	.12	1.2	2
Belgium	1930-1935	.3	.24	1.5	1.6
	1936-1940	.3	.21	1.3	1.5
	1941-1945	.18	.19	1.6	1.6
	1946-1950	.19	.17	1.5	1.6
	Total	.24	.2	1.5	1.6
Bulgaria	1930-1935	.11	.047	1.2	1.7
	1936-1940	.17	.045	1.2	1.7
	1941-1945	.13	.025	1.3	1.9
	1946-1950	.13	.04	1.4	1.9
	Total	.13	.039	1.3	1.8
Czech Republic	1930-1935	.26	.081	1	1.8
	1936-1940	.14	.12	1.4	1.6
	1941-1945	.13	.063	1.4	1.9
	1946-1950	.1	.068	1.6	1.9
	Total	.16	.083	1.4	1.8
Georgia	1930-1935	.38	.29	.78	1.3
	1936-1940	.31	.23	1	1.5
	1941-1945	.29	.21	.93	1.6
	1946-1950	.29	.16	1	1.7
	Total	.32	.22	.94	1.5
Germany	1930-1935	.32	.31	1	1.2
	1936-1940	.4	.3	.79	1.1
	1941-1945	.29	.33	1.1	.99
	1946-1950	.26	.27	1	1.1
	Total	.32	.3	.98	1.1
Lithuania	1930-1935	.29	.39	.82	.73
	1936-1940	.25	.19	.95	1.4
	1941-1945	.26	.18	1	1.3
	1946-1950	.22	.12	1	1.6
	Total	.26	.22	.95	1.3

Tab. A3: Continuation

Country	Cohort	Proportion of childless		Cohort fertility rate	
		High- educated	Low- educated	High- educated	Low- educated
The Netherlands	1930-1935	.29	.18	.28	2.2
	1936-1940	.23	.13	1.6	2
	1941-1945	.27	.098	1.2	2
	1946-1950	.28	.12	1.2	1.9
	Total	.27	.13	1.4	2
Poland	1930-1935	.15	.11	1.5	2.3
	1936-1940	.14	.092	1.3	2.2
	1941-1945	.14	.077	1.4	2.2
	1946-1950	.22	.081	1.2	2.1
	Total	.16	.091	1.4	2.2
Romania	1930-1935	.21	.17	.78	1.4
	1936-1940	.4	.13	.53	1.7
	1941-1945	.23	.12	1	1.9
	1946-1950	.13	.099	1.3	1.9
	Total	.25	.13	.92	1.7
Russia	1930-1935	.12	.15	1.4	1.5
	1936-1940	.15	.094	1.3	1.7
	1941-1945	.12	.078	1.5	1.7
	1946-1950	.1	.062	1.6	1.9
	Total	.12	.095	1.4	1.7
Sweden	1930-1935	.14	.12	2	2
	1936-1940	.15	.067	1.9	2.2
	1941-1945	.14	.098	1.9	2
	1946-1950	.11	.11	2	2
	Total	.13	.1	1.9	2.1

Source: own calculation based on the pooled prospective dataset.

Tab. A4: Proportion of high- and low-educated women in a partnership

Country	Cohort	Proportion of women with a partner	
		High-educated	Low-educated
Austria	1930-1935	.87	.77
	1936-1940	.79	.86
	1941-1945	.77	.85
	1946-1950	.72	.82
	Total	.77	.83
Belgium	1930-1935	.8	.83
	1936-1940	.81	.84
	1941-1945	.81	.88
	1946-1950	.83	.89
	Total	.82	.86
Bulgaria	1930-1935	.75	.8
	1936-1940	.84	.85
	1941-1945	.85	.93
	1946-1950	.87	.93
	Total	.84	.89
Czech Republic	1930-1935	.75	.85
	1936-1940	.83	.86
	1941-1945	.86	.89
	1946-1950	.86	.91
	Total	.84	.88
Georgia	1930-1935	.51	.63
	1936-1940	.66	.75
	1941-1945	.7	.78
	1946-1950	.69	.82
	Total	.66	.75
Germany	1930-1935	.6	.7
	1936-1940	.72	.82
	1941-1945	.86	.86
	1946-1950	.83	.88
	Total	.78	.82

Tab. A4: Continuation

Country	Cohort	Proportion of women with a partner	
		High-educated	Low-educated
Lithuania	1930-1935	.36	.54
	1936-1940	.62	.63
	1941-1945	.7	.72
	1946-1950	.71	.78
	Total	.66	.68
The Netherlands	1930-1935	.59	.81
	1936-1940	.73	.89
	1941-1945	.78	.87
	1946-1950	.76	.88
	Total	.74	.87
Poland	1930-1935	.8	.78
	1936-1940	.82	.83
	1941-1945	.77	.86
	1946-1950	.77	.87
	Total	.78	.84
Romania	1930-1935	.89	.8
	1936-1940	.86	.85
	1941-1945	.83	.9
	1946-1950	.91	.93
	Total	.88	.88
Russia	1930-1935	.67	.59
	1936-1940	.66	.65
	1941-1945	.7	.72
	1946-1950	.75	.78
	Total	.7	.69
Sweden	1930-1935	.79	.81
	1936-1940	.84	.87
	1941-1945	.8	.86
	1946-1950	.83	.83
	Total	.82	.85

Source: own calculation based on the pooled prospective dataset.

Tab. A5: Proportion of women across mating patterns

Country	Cohort	No partner	Homogamous high	hypogamy	Homogamous low	hypergamy
Austria	1930-1935	.11	.062	.066	.66	.096
	1936-1940	.097	.058	.047	.69	.11
	1941-1945	.12	.089	.062	.62	.1
	1946-1950	.14	.085	.059	.6	.12
	Total	.12	.073	.059	.64	.11
Belgium	1930-1935	.19	.08	.053	.59	.087
	1936-1940	.17	.11	.062	.55	.11
	1941-1945	.15	.14	.066	.53	.12
	1946-1950	.13	.16	.082	.52	.11
	Total	.16	.12	.066	.55	.11
Bulgaria	1930-1935	.17	.067	.022	.66	.088
	1936-1940	.13	.092	.038	.66	.081
	1941-1945	.055	.095	.065	.74	.041
	1946-1950	.078	.1	.068	.69	.062
	Total	.11	.089	.048	.69	.068
Czech Republic	1930-1935	.13	.02	.024	.024	.073
	1936-1940	.14	.037	.037	.71	.081
	1941-1945	.099	.058	.049	.73	.062
	1946-1950	.086	.033	.05	.76	.071
	Total	.11	.037	.041	.74	.072
Georgia	1930-1935	.4	.068	.037	.44	.056
	1936-1940	.28	.088	.054	.54	.044
	1941-1945	.25	.13	.071	.51	.044
	1946-1950	.23	.16	.061	.46	.094
	Total	.29	.11	.056	.49	.059
Germany	1930-1935	.2	.052	.023	.58	.15
	1936-1940	.14	.058	.031	.58	.19
	1941-1945	.12	.058	.055	.54	.17
	1946-1950	.12	.13	.042	.52	.19
	Total	.14	.089	.038	.55	.18
Lithuania	1930-1935	.48	.019	.005	.46	.041
	1936-1940	.36	.055	.038	.52	.033
	1941-1945	.28	.079	.076	.51	.062
	1946-1950	.22	.065	.076	.58	.056
	Total	.33	.054	.049	.52	.048

Tab. A5: Continuation

Country	Cohort	No partner	Homogamous high	hypogamy	Homogamous low	hypergamy
The Netherlands	1930-1935	.17	.066	.032	.62	.1
	1936-1940	.14	.084	.05	.59	.13
	1941-1945	.13	.14	.032	.54	.15
	1946-1950	.13	.14	.053	.55	.13
	Total	.14	.11	.042	.58	.13
Poland	1930-1935	.19	.036	.027	.7	.054
	1936-1940	.13	.057	.046	.69	.071
	1941-1945	.12	.038	.047	.74	.056
	1946-1950	.12	.064	.04	.72	.053
	Total	.14	.049	.04	.71	.058
Romania	1930-1935	.16	.026	0	.78	.031
	1936-1940	.13	.034	.0096	.82	.014
	1941-1945	.096	.052	.0053	.8	.05
	1946-1950	.062	.052	.027	.81	.046
	Total	.11	.041	.01	.8	.035
Russia	1930-1935	.26	.1	.049	.54	.042
	1936-1940	.19	.17	.092	.47	.072
	1941-1945	.18	.22	.11	.44	.052
	1946-1950	.16	.21	.094	.43	.11
	Total	.2	.18	.085	.47	.069
Sweden	1930-1935	.25	.055	.054	.56	.081
	1936-1940	.17	.09	.08	.57	.096
	1941-1945	.19	.092	.093	.53	.099
	1946-1950	.21	.099	.094	.5	.099
	Total	.2	.084	.08	.54	.094

Source: own calculation based on the pooled prospective dataset.

Tab. A6: Educational production rate of high- and low-educated women producing high- and low-educated offspring

Country	Cohort	Educational production rate of high-educated offspring		Educational production rate of low-educated offspring	
		High-educated women	Low-educated women	High-educated women	Low-educated women
Austria	1930-1935	1.1	.48	.3	1.9
	1936-1940	.71	.61	.3	1.7
	1941-1945	1.2	.58	.42	1.5
	1946-1950	.82	.47	.43	1.5
	Total	.95	.54	.36	1.6
Belgium	1930-1935	1.5	.6	.69	.96
	1936-1940	1.4	.67	.39	.91
	1941-1945	1.4	.81	.31	.9
	1946-1950	1.5	.87	.22	.83
	Total	1.4	.74	.4	.9
Bulgaria	1930-1935	1.1	.36	.3	1.2
	1936-1940	.84	.39	.55	1.2
	1941-1945	1	.44	.36	1.4
	1946-1950	.93	.43	.56	1.4
	Total	.98	.41	.44	1.3
Czech Republic	1930-1935	.64	.4	.54	1.5
	1936-1940	.62	.34	.95	1.4
	1941-1945	.69	.36	.83	1.5
	1946-1950	.56	.3	1.1	1.7
	Total	.63	.35	.85	1.5
Georgia	1930-1935	.74	.41	.25	.99
	1936-1940	1.1	.43	.35	1.2
	1941-1945	.98	.57	.25	1.2
	1946-1950	1.1	.58	.27	1.3
	Total	.99	.5	.28	1.2
Germany	1930-1935	1	.52	.39	1.1
	1936-1940	.95	.45	.34	1
	1941-1945	.96	.43	.42	.95
	1946-1950	.47	.34	.82	1
	Total	.85	.44	.49	1
Lithuania	1930-1935	.66	.27	.32	.9
	1936-1940	.87	.4	.26	1.1
	1941-1945	.74	.34	.33	1
	1946-1950	.58	.45	.63	1.1
	Total	.71	.37	.39	1

Tab. A6: Continuation

Country	Cohort	Educational production rate of high-educated offspring		Educational production rate of low-educated offspring	
		High-educated women	Low-educated women	High-educated women	Low-educated women
The Netherlands	1930-1935	1.8	.92	.64	1.6
	1936-1940	1.5	.98	.46	1.4
	1941-1945	1.2	.89	.44	1.2
	1946-1950	.88	.79	.65	1.2
	Total	1.3	.9	.55	1.3
Poland	1930-1935	.72	.39	.45	1.7
	1936-1940	1.2	.43	.23	1.7
	1941-1945	1.1	.56	.43	1.6
	1946-1950	1	.65	.45	1.5
	Total	1	.51	.39	1.6
Romania	1930-1935	-	.17	-	1.4
	1936-1940	.69	.2	.12	1.6
	1941-1945	1.2	.32	.04	1.6
	1946-1950	1.2	.28	.25	1.7
	Total	1	.24	.14	1.6
Russia	1930-1935	1.1	.76	.3	.88
	1936-1940	1.1	.71	.37	.88
	1941-1945	1	.68	.41	.99
	1946-1950	1.1	.65	.47	1.2
	Total	1.1	.7	.39	.98
Sweden	1930-1935	1	.33	.82	1.1
	1936-1940	.91	.59	.79	1.6
	1941-1945	.96	.55	.83	1.4
	1946-1950	1.1	.64	.74	1.3
	Total	1	.53	.8	1.3

Source: own calculation based on the pooled prospective and retrospective datasets.

Tab. A7: Decomposition of educational differences in rates of production

Country	Cohort	Educational production rate of high-educated offspring			Educational production rate of low-educated offspring		
		fertility	Direct inheritance	mating	fertility	Direct inheritance	mating
Austria	1930-1935	-.38	1	-.031	.38	1	.21
	1936-1940	-.25	.48	-.13	.29	.48	.66
	1941-1945	-.12	.79	-.056	.11	.79	.17
	1946-1950	-.13	.55	-.07	.19	.55	.3
	Total	-.22	.71	-.072	.24	.71	.33
Belgium	1930-1935	.2	.22	.46	-.076	.22	.13
	1936-1940	.087	.39	.29	-.041	.39	.18
	1941-1945	.039	.34	.18	-.0072	.34	.26
	1946-1950	-.013	.45	.21	.0042	.45	.16
	Total	.078	.35	.29	-.03	.35	.18
Bulgaria	1930-1935	-.079	.47	.36	.083	.47	.33
	1936-1940	-.07	.37	.15	.054	.37	.19
	1941-1945	-.14	.71	.032	.11	.71	.26
	1946-1950	-.069	.4	.17	.1	.4	.33
	Total	-.089	.49	.18	.086	.49	.28
Czech Republic	1930-1935	-.11	.093	.26	.19	.093	.64
	1936-1940	-.021	.098	.2	.062	.098	.28
	1941-1945	-.055	.14	.25	.23	.14	.31
	1946-1950	-.038	.21	.084	.12	.21	.23
	Total	-.055	.14	.2	.15	.14	.36
Georgia	1930-1935	-.12	.42	.035	.085	.42	.23
	1936-1940	.06	.61	-.012	-.071	.61	.34
	1941-1945	-.35	.44	.44	.29	.44	.25
	1946-1950	-.15	.7	.0013	.085	.7	.21
	Total	-.14	.54	.086	.098	.54	.26
Germany	1930-1935	-.071	.41	.15	.069	.069	.23
	1936-1940	-.045	.54	.0038	-.024	.54	.19
	1941-1945	.069	.34	.12	-.065	.34	.26
	1946-1950	.049	-.0014	.079	-.11	-.0014	.31
	Total	.00017	.32	.088	-.032	.32	.25
Lithuania	1930-1935	.015	.5	-.13	-.02	.5	.1
	1936-1940	-.15	.54	.086	.16	.54	.18
	1941-1945	-.088	.48	.0028	.066	.48	.14
	1946-1950	-.04	.13	.028	.077	.13	.23
	Total	-.067	.41	-.0027	.071	.41	.17

Tab. A7: Continuation

Country	Cohort	Educational production rate of high-educated offspring			Educational production rate of low-educated offspring		
		fertility	Direct inheritance	mating	fertility	Direct inheritance	mating
The Netherlands	1930-1935	.032	.24	.57	.03	.24	.67
	1936-1940	-.062	.5	.058	.14	.5	.31
	1941-1945	-.065	.21	.18	-.054	.21	.6
	1946-1950	-.087	.076	.1	.059	.076	.41
	Total	-.045	.26	.23	.045	.26	.5
Poland	1930-1935	-.25	.44	.13	.31	.44	.5
	1936-1940	-.34	1	.05	.33	1	.14
	1941-1945	-.15	.64	.082	.23	.64	.34
	1946-1950	-.13	.39	.13	.18	.39	.48
	Total	-.21	.63	.098	.26	.63	.36
Romania	1930-1935	-	-	-	-	-	-
	1936-1940	-.42	.67	.25	.39	.67	.44
	1941-1945	-.042	.86	.093	-.13	.86	.84
	1946-1950	-.041	.8	.18	.078	.8	.53
	Total	-.17	.78	.17	.11	.78	.61
Russia	1930-1935	-.21	.35	.35	.083	.35	.15
	1936-1940	-.14	.3	.18	.091	.3	.13
	1941-1945	-.033	.35	.045	.046	.35	.18
	1946-1950	-.061	.43	.081	.044	.43	.24
	Total	-.11	.36	.14	.066	.36	.17
Sweden	1930-1935	.067	.44	.18	-.092	.44	-.08
	1936-1940	.0086	.37	-.061	-.0077	.37	.48
	1941-1945	.057	.38	-.03	-.078	.38	.27
	1946-1950	.052	.42	.016	-.06	-.06	.17
	Total	.046	.4	.026	-.06	.4	.21

Source: own calculation based on the pooled prospective and retrospective datasets.

B1 Description of three-way counterfactual decomposition

In their analysis of inequality in educational production rates (EPRs) between higher- and lower-educated women, *Skopek and Leopold (2020)* proposed a method to decompose the inequality into the effect of two components – fertility and attainment (the “mobility effect” in their terms). The method builds on the idea of calculating counterfactual EPRs, sequentially attributing the fertility and attainment rates of one educational group to the other. Specifically, assuming that EPR is a function of two effects – fertility (F) and attainment (A) – the inequality in EPR of children attaining education j between women with education i and women with education i' can be decomposed as follows

$$\begin{aligned} \Delta^j &= r_{ji} - r_{ji'} = r_j(F_i, A_i) - r_j(F_i, A_i) & (1) \\ &= \frac{1}{2} \left(\underbrace{r_j(F_i, A_i) - r_j(F_{i'}, A_i) + r_j(F_i, A_{i'}) - r_j(F_{i'}, A_{i'})}_{\Delta_F} \right) \\ &\quad + \frac{1}{2} \left(\underbrace{r_j(F_i, A_i) - r_j(F_i, A_{i'}) + r_j(F_{i'}, A_i) - r_j(F_{i'}, A_{i'})}_{\Delta_A} \right) = \Delta_F + \Delta_A \end{aligned}$$

where $r_{ji} = r_j(F_i, A_i)$ and $r_{ji'} = r_j(F_{i'}, A_{i'})$ represent factual EPRs for women with education i and i' respectively, and terms $r_j(\mathbf{F}_i, A_i)$, $r_j(\mathbf{F}_i, A_{i'})$, $r_j(F_i, \mathbf{A}_i)$, $r_j(F_{i'}, \mathbf{A}_i)$ represent counterfactual quantities. In turn, Δ_F and Δ_A correspond to the average “effects” of swapping fertility and attainment rates for women in comparison (“effect” in this case meaning the difference between the factual EPR and the respective counterfactual one).

In the example above, however, only two aspects of EPR are accounted for, whereas in our extended model we consider three:

$$r_{ji} = \sum_m \left(\frac{P(M|I)}{M} \cdot \frac{E(F|I, M)}{F} \cdot \frac{P(J|I, M)}{A} \right) = r(M, F, A) \quad (2)$$

In other words, we assume EPR to be a function of three aspects – mating, fertility, and attainment.

This makes decomposition a bit more challenging, making it three-fold instead of two-fold. It builds on a similar logic (i.e., calculating differences using counterfactual states), but the unfolding involves several additional steps. First, we use the logic of Equation 1 to decompose the difference Δ^j as follows:

$$\begin{aligned}
 \Delta^j &= r_j(M_i, F_i, A_i) - r_j(M_{i'}, F_{i'}, A_{i'}) & (3) \\
 &= \frac{1}{2} \left(\underbrace{\left(r_j(M_i, F_i, A_i) - r_j(M_{i'}, F_i, A_i) \right)}_{\Delta_M} + \left(r_j(M_i, F_{i'}, A_{i'}) - r_j(M_{i'}, F_{i'}, A_{i'}) \right) \right) \\
 &\quad + \frac{1}{2} \left(\underbrace{\left(r_j(M_i, F_i, A_i) - r_j(M_i, F_{i'}, A_{i'}) \right)}_{\Delta_{FA}} + \left(r_j(M_{i'}, F_i, A_i) - r_j(M_{i'}, F_{i'}, A_{i'}) \right) \right) \\
 &= \Delta_M + \Delta_{FA}
 \end{aligned}$$

In the equation above, Δ_M is the average of the effect assuming a counterfactual distribution of mating patterns for group i and i' keeping factual fertility and attainment rates. In turn, Δ_{FA} is the average of the effect of swapping both the fertility and attainment rates keeping factual mating patterns.

Using a similar logic we obtain:

$$\Delta^j = \Delta_F + \Delta_{MA} \quad (4)$$

$$\Delta^j = \Delta_A + \Delta_{MF} \quad (5)$$

Accordingly, combining Equations 3-5:

$$\Delta^j = \frac{1}{3} (\Delta_M + \Delta_F + \Delta_A + \Delta_{FA} + \Delta_{MA} + \Delta_{MF}) \quad (6)$$

The next step involves breaking the joint effects Δ_{FA} , Δ_{MA} , and Δ_{MF} into the fractions that represent individual effects. This step exploits the idea that the joint effects can be redefined as the average of the differences in factual EPRs for different levels of the counterfactual variable. Using the case of Δ_{FA} as an example:

$$\Delta_{FA} = \frac{1}{2} \left(\underbrace{\left(r_j(M_i, F_i, A_i) - r_j(M_i, F_{i'}, A_{i'}) \right)}_{\Delta_{|M_i}} \right) + \frac{1}{2} \left(\underbrace{\left(r_j(M_{i'}, F_i, A_i) - r_j(M_{i'}, F_{i'}, A_{i'}) \right)}_{\Delta_{|M_{i'}}} \right) = \frac{1}{2} \Delta_{|M_i} + \frac{1}{2} \Delta_{|M_{i'}} \quad (7)$$

where $\Delta_{|M_i}$ and $\Delta_{|M_{i'}}$ can correspond to differences in EPRs between i and i' for counterfactual states assuming (i.e., holding constant) the mating pattern distribution of i and i' respectively.

In turn, the terms $\Delta_{|M_i}$ and $\Delta_{|M_{i'}}$ can be further decomposed using the logic of Equation 2. Using $\Delta_{|M_i}$ as an example:

$$\begin{aligned}
 \Delta_{|M_i} &= r_j(M_i, F_i, A_i) - r_j(M_i, F_{i'}, A_{i'}) & (8) \\
 &= \frac{1}{2} \left(\underbrace{\left(r(M_i, F_i, A_i) - r(M_i, F_i, A_{i'}) \right)}_{\Delta_{A|M_i}} + \underbrace{\left(r(M_i, F_{i'}, A_i) - r(M_i, F_{i'}, A_{i'}) \right)}_{\Delta_{F|M_i}} \right) \\
 &= \Delta_{A|M_i} + \Delta_{F|M_i}
 \end{aligned}$$

where $\Delta_{A|M_i}$ is the effect of swapping just the attainment rates and $\Delta_{F|M_i}$ is the effect of swapping just the fertility rates, for the assumed mating distribution of women with education i . Accordingly:

$$\Delta_{FA} = \frac{1}{2}(\Delta_{F|M_i} + \Delta_{F|M_{i'}}) + \frac{1}{2}(\Delta_{A|M} + \Delta_{A|M_{i'}}) \quad (9)$$

$$\Delta_{MA} = \frac{1}{2}(\Delta_{M|F_i} + \Delta_{M|F_{i'}}) + \frac{1}{2}(\Delta_{A|F_i} + \Delta_{A|F_{i'}}) \quad (10)$$

$$\Delta_{MF} = \frac{1}{2}(\Delta_{M|A_i} + \Delta_{M|A_{i'}}) + \frac{1}{2}(\Delta_{F|A_i} + \Delta_{F|A_{i'}}) \quad (11)$$

Thus, the joint effect of any two components (e.g., fertility and attainment in the case of Δ_{FA}) can also be redefined as the average of its single constituent effects calculated for every counterfactual of the remaining component (i.e., mating patterns in the example used).

Finally, combining Equation 6 and Equations 9-11, the difference in factual EPRs can be shown to unfold as follows:

$$\begin{aligned} \Delta^j &= \frac{1}{3}(\Delta_M + \Delta_F + \Delta_A + \Delta_{FA} + \Delta_{MA} + \Delta_{MF}) \quad (12) \\ &= \frac{1}{3}(\underbrace{\Delta_M + \Delta_{M|F_i} + \Delta_{M|F_{i'}} + \Delta_{M|A_i} + \Delta_{M|A_{i'}}}_{\bar{\Delta}_M}) + \frac{1}{3}(\underbrace{\Delta_F + \Delta_{F|M_i} + \Delta_{F|M_{i'}} + \Delta_{F|A_i} + \Delta_{F|A_{i'}}}_{\bar{\Delta}_F}) \\ &\quad + \frac{1}{3}(\underbrace{\Delta_A + \Delta_{A|M_i} + \Delta_{A|M_{i'}} + \Delta_{A|F_i} + \Delta_{A|F_{i'}}}_{\bar{\Delta}_A}) = \bar{\Delta}_M + \bar{\Delta}_F + \bar{\Delta}_A \end{aligned}$$

where $\bar{\Delta}_M$, $\bar{\Delta}_F$ and $\bar{\Delta}_A$ represent the average effects of swapping mating patterns, fertility, and attainment rates respectively.

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